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WAVE FORCES ON BREAKWATERS

By Robert Y. Hudson, M. ASCE

HYDRAULICS DIVISION

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PAPERS

WAVE FORCES ON BREAKWATERS

By Robert Y. Hudson,[1] M. ASCE

Synopsis

This paper reviews the common theories for determining the magnitude and distribution of wave forces on vertical walls and sloping, rubble-mound breakwaters and compares these theories, selecting the best available for use. The results of the better theories are compared with experimental data, and an outline is given of experimental investigations considered necessary to obtain data sufficient for the design of economical and safe breakwaters.

Introduction

The two principal types of breakwaters are: (*a*) Vertical-wall structures; and (*b*) those with sloping faces constructed of rubble. Each of these types offers certain advantages, depending primarily on location and exposure to wave action. Because of the complexity of the phenomena, whereby waves exert forces on breakwaters, it is difficult to arrive at a rational basis for determining the magnitude and distribution of forces with sufficient accuracy to insure that both safe and economical designs are obtained. Design engineers make use of theoretical and empirical formulas, several of which are explained in this paper, but there is no formula available that has won the complete confidence of the engineering profession. It has become possible to calculate with reasonable accuracy the height, length, and direction of design waves as a function of fetch and wind velocity, duration, and direction.[2] It has also become possible to chart selected design waves from deep water into shallow water and determine wave characteristics at the location of proposed breakwaters.[3] There is an urgent need for methods by which breakwaters may be designed with an accuracy compatible with the accuracy with which

Note.—Written comments are invited for publication; the last discussion should be submitted by July 1, 1952.

[1] Hydr. Eng.; Chf., Wave Action Section, Waterways Experiment Station, Corps of Engrs., U. S. Army, Vicksburg, Miss.

[2] "Wind, Sea, and Swell: Theory of Relations for Forecasting," by H. U. Sverdrup and W. H. Munk, Hydrographic Office, U. S. Navy, Washington, D. C., 1947.

[3] "Breakers and Surf, Principles in Forecasting," Hydrographic Office, U. S. Navy, Washington, D. C., 1944.

design-wave characteristics can be determined. Although the science of waves and wave action on breakwaters is not exactly in its infancy (some of the best mathematicians and engineers having worked on it since about 1800), the design of breakwaters is yet to be placed on a sound scientific basis in which known safety factors can be selected. The purpose of this paper, therefore, is threefold: (1) To review the most important theories pertaining to wave forces on breakwaters; (2) to compare these theories and select the best available for use at the present time; and (3) to compare the results of the better theories with experimental data and point out the type of additional information required to satisfy the demands of design engineers.

Notation.—Letter symbols are defined where they first appear and are assembled alphabetically for convenience of reference in Appendix I.

Vertical-Wall Breakwaters

Vertical-wall breakwaters are usually considered most efficient when the structure is to be situated in water of considerable depth or when stone for construction of a rubble mound is not available at economical prices. If water is of sufficient depth to prevent the breaking of waves (that is, the depth of water below still-water level (d) is greater than the wave height (H) from trough to crest) and if wave heights are relatively small, standing waves are formed with accompanying pressures for which fairly accurate theoretical formulas exist giving their overturning effect on the structure. However, as the waves increase in magnitude, available evidence indicates that pressures become larger than predicted by available theory. Also, if waves break on a vertical wall, localized pressures may be produced that are several times greater than those resulting from nonbreaking waves.

Wave Theories.—The bases upon which some of the most important theories of wave force on vertical breakwaters are founded are the mathematical works of Franz V. Gerstner,[4] and Barre de St. Venant and A. Flamant.[5] Mr. Gerstner, in 1802, obtained an exact solution of the equations of motion for deep-water waves. However, his solution is based on geometrical considerations rather than the fundamental equations of motion. Also, the motion is rotational, and the waves cannot be generated by natural forces. The parametric equations that define the Gerstner wave form referred to rectangular axes x and y, and in terms of the parameter x_o, are:

$$x = x_o - r \sin \frac{2\pi x_o}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

$$y = y_o + h_z + r \cos \frac{2\pi x_o}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1b)$$

in which x is the abscissa and y is the ordinate of a particle in wave motion; x_o and y_o are the abscissa and ordinate, respectively, of a particle at rest; r is the horizontal radius of the orbital ellipse; L is the wave length; and h_z is the distance of a particle's orbital axis above the plane of rest.

[4] "Theorie der Wellen," by Franz V. Gerstner, *Abhandlungen der königlichen Bohmischen Gesellschaft der Wissenschaften*, Prague, Czechoslovakia, 1802.

[5] "De la Houle et du Clapotis," by Barre de St. Venant and A. Flamant, *Annales des ponts et chaussees*, Paris, France, 1888.

These equations define trochoids with the y-axis through a crest and the x-axis on the still-water level (SWL). The curves are generated by a point P that lies at a distance r from the center of a circle of circumference L that rolls on a line at a distance $L/(2\pi)$ above the orbital axis. The tracing point P describes a wave form with the orbital axis $y = y_o + h_z$. The undisturbed level of this wave form is $y = y_o$. The term y_o is always negative or equal to zero. The method by which the wave form may be determined geometrically and the relations between the variables described are shown by Fig. 1.

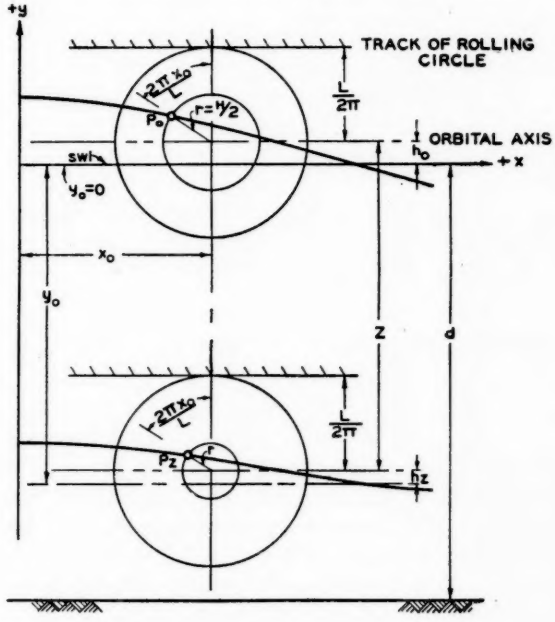

FIG. 1.—GEOMETRICAL DETERMINATION OF WAVE FORM

The following relations are in strict accord with the general equations of hydrodynamics:

$$h_z = \frac{\pi r^2}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2a)$$

When $y_o = 0$, radius $r = H/2$, and Eq. 2a becomes:

$$h_o = \frac{\pi H^2}{4 L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2b)$$

in which h_o is the distance of the surface particles' orbital axis above SWL. Eq. 2b is the value of h_z corresponding to the free surface. Also:

$$r = \frac{H}{2} e^{2\pi z/L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

in which e is the base of natural logarithms and z, as defined in Fig. 1, is:

$$z = y_o + h_z - h_o \quad \ldots\ldots (3b)$$

Eq. 3*a* is not an explicit formula for r, since the value of h_z in Eq. 3*b* depends on r, but it can be solved by successive approximations. However, since the term $h_z - h_o$ in Eq. 3*b* is small, approximately:

$$r = \frac{H}{2} e^{2\pi y_o/L} \quad \ldots\ldots (4)$$

The orbits of particles in Gerstner waves are circles of radius r, and these particles describe their orbits with constant angular velocity $2\pi/T$. The orbital velocity (V_z) of a particle originally at rest at depth y_o is:

$$V_z = \frac{2\pi r}{T} \quad \ldots\ldots (5a)$$

in which T is the wave period. At the free surface, the orbital velocity V_o is:

$$V_o = \frac{\pi H}{T} \quad \ldots\ldots (5b)$$

The wave celerity (C) is defined by the equation:

$$C^2 = \frac{g L}{2\pi} \quad \ldots\ldots (6)$$

in which g is the acceleration of gravity.

All of the trochoids generated by rolling circles with centers at different depths ($y_o + h_z$) below SWL become instantaneous stream lines along which the pressure is constant and equal to the product of y and to γ, the specific weight of the liquid. In other words, the pressure along any stream line, the particles of which were originally at rest at a depth y_o, is equal to the hydrostatic pressure corresponding to the depth y_o.

Although the Gerstner theory affords a mathematically exact solution to a type wave that approximates observed wave characteristics in deep water, it does not describe the characteristics of waves in shallow water. In 1888 Mr. de St. Venant and Mr. Flamant[5] generalized the Gerstner theory of the orbital motion by assuming the particle orbits to be ellipses. They obtained the following equations for the wave form:

$$x = x_o - r \sin \frac{2\pi x_o}{L} \quad \ldots\ldots (7a)$$

and

$$y = y_o + h_z + r' \cos \frac{2\pi x_o}{L} \quad \ldots\ldots (7b)$$

in which r' is the vertical radius of orbital ellipse (in deep water $r = r'$). For these waves,

$$h_z = \frac{\pi r r'}{L} \quad \ldots\ldots (8)$$

The stream lines described by Eq. 7 have been called elliptical trochoids, although the curves cannot be determined by a rolling motion as in the case of simple trochoids. The orbital motion defined by these equations does not conform exactly to the continuity equation, as do those for the Gerstner waves. However, the following equations, which define orbital radii, are approximate solutions of the continuity equation:

$$r = (H/2)\frac{\cosh\frac{2\pi}{L}(d + y_o)}{\sinh\frac{2\pi d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9a)$$

$$r' = (H/2)\frac{\sinh\frac{2\pi}{L}(d + y_o)}{\sinh\frac{2\pi d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9b)$$

These equations satisfy the condition that $r' = H/2$ when $y_o = 0$, and $r' = 0$ when $y_o = -d$. Other relations between the variables of the elliptical trochoid theory for waves in water of finite depth, according to Messrs. de St. Venant and Flamant, are:

a. At the free surface, when $y_o = 0$ and $r' = H/2$, Eq. 8 becomes

$$h_o = \frac{\pi H^2}{4L}\coth\frac{2\pi d}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

b. The particles describe elliptical orbits with an angular velocity that is not constant. The orbital crest velocity at any depth (V_c) is $2\pi r/T$, and for a particle initially at rest at depth y_o, the crest velocity of the particle is

$$V_{zc} = \frac{\pi H}{T}\frac{\cosh\frac{2\pi}{L}(d + y_o)}{\sinh\frac{2\pi d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11a)$$

At the free surface $y_o = 0$, and $V_{oc} = \frac{\pi H}{T}\coth\frac{2\pi d}{L}$ ..... (11*b*)

c. The wave celerity is $C^2 = \frac{gL}{2\pi}\tanh\frac{2\pi d}{L}$ ..... (12)

which becomes $C^2 = \frac{gL}{2\pi}$ as d/L approaches infinity, and $C^2 = gd$ as d/L approaches zero.

d. The pressure along any elliptically trochoidal stream line is approximately constant and is equal to:

$$\frac{p}{\gamma} = z + \frac{gT^2}{2L^2}\left[\frac{H^2}{4} - \left(\frac{2\pi rL}{gT^2}\right)^2\right] - \left(\frac{2\pi L}{gT^2} - r'\right)\cos\frac{2\pi x_o}{L} \quad \ldots (13)$$

in which p is the pressure intensity.

Wave-Pressure Theories.—Thomas Stevenson (1842–1858)[6] and D. D. Gaillard (1890–1903)[7] made extensive dynamometer measurements of wave forces on breakwaters and shoreline structures. However, the accuracy of their measuring equipment was not adequate, and the manner in which the data were obtained did not permit accurate determination of the magnitude and distribution of the pressures or the correlation of pressures with wave dimensions.

In 1890 Mr. L. d'Auria[8] calculated the pressure of waves on a vertical wall by applying the principle of "quantity of motion" (momentum). Dividing the quantity of motion, $H\,L\,C/(2\,g)$, by the duration of motion, $L/(2\,C)$, he obtained the mean pressure,

$$\frac{p}{\gamma} = \frac{H\,C^2}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

that was assumed to be equally distributed over the height of the wave, presumably about the SWL. To this pressure was added the hydrostatic pressure corresponding to the depth of water at the crest position of the wave. The sum of these pressures was taken as the maximum on the structure acting shoreward. If there is water on both seaward and shoreward sides of the structure and if there exists no wave action on the shoreward side, the net unbalanced overturning pressure is obtained by subtracting from the sum of the two pressures mentioned the hydrostatic pressure corresponding to the SWL depth.

A method somewhat similar to that of Mr. d'Auria but utilizing to some extent the wave theories of Messrs. Gerstner and de St. Venant and Mr. Flamant, was proposed in 1926 by Jorge Lira Orrego.[9] The Lira theory is explained as follows: At the instant when the highest point of the wave crest comes in contact with the vertical wall, there is a pressure of the static type because of the height of the wave crest above SWL, and a dynamic pressure, extending from the wave crest to the bottom, caused by the wave particles that at this instant are at the upper part of their elliptical orbits and have the corresponding orbital velocities given by Eq. 11. Mr. Lira assumes the height of the wave crest above SWL to be $H/2 + h_o$, in which from Eqs. 2 and 9*a*,

$$h_o = \frac{\pi\,H^2}{4\,L} \coth^2 \frac{2\,\pi\,d}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

The hydrostatic SWL pressure is deducted from the sum of the static and dynamic pressures to obtain the net pressure acting shoreward if the structure has water on its inner side without wave action. The dynamic pressure created by the cresting wave, according to Mr. Lira, is given by the equation:

$$\frac{p_z}{\gamma} = \frac{k\,V_c^2}{2\,g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

[6] "The Design and Construction of Harbours," by Thomas Stevenson, Adam Blackford, and Charles Blackford, 3rd Ed., Edinburgh, Scotland, 1886.

[7] "Wave Action in Relation to Engineering Structures," by D. D. Gaillard, U. S. Govt. Printing Office, Washington, D. C., 1904.

[8] "On the Force of Impact of Waves and the Stability of the Superstructure of Breakwaters," by L. d'Auria, *Journal*, Franklin Institute, Vol. 130, 1890, p. 373.

[9] "Breakwaters or Jetties in Tideless Seas," by Jorge Lira Orrego, *Report No. 29*, XIVth International Congress of Navigation, Cairo, Egypt, 1926.

The coefficient k was assumed to have a value of 4. By substituting this value and the expression for V_c (Eq. 11) in Eq. 16, the equation for dynamic pressure is obtained:

$$\frac{p_z}{\gamma} = \frac{2\pi^2 H^2}{g T^2} \frac{\cosh^2 \frac{2\pi}{L}(d + y_o)}{\sinh^2 \frac{2\pi d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17a)$$

When $y_o = 0$, at the free surface,

$$\frac{p_o}{\gamma} = \frac{2\pi^2 H^2}{g T^2} \coth^2 \frac{2\pi d}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17b)$$

When $y_o = -d$, at the bottom,

$$\frac{p_d}{\gamma} = \frac{2\pi^2 H^2}{g T^2 \sinh^2 \frac{2\pi d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17c)$$

In 1938 Ramon Iribarren Cavanilles[10] modified portions of the de St. Venant and Flamant theory and obtained equations for the static and dynamic pressures caused by waves on a vertical wall. The static pressures were calculated, based on those in a trochoidal movement at any depth, and are less than those of Mr. Lira. The Iribarren dynamic-pressure equations introduce the wave celerity instead of orbital velocity, as did those of Mr. d'Auria, and the resulting pressures are greater than those found by the Lira formula. Mr. Iribarren determines the height of the wave axis above SWL (h_o) to be the same as that given by Messrs. de St. Venant and Flamant (Eq. 10). The dynamic pressures $\left(\frac{p}{\gamma}\right)_{dy}$ assumed to act at orbital crests at distances from the SWL of

$$y = y_o + h_z + H \frac{\sinh \frac{2\pi}{L}(d + y_o)}{\sinh \frac{2\pi d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

are given by the equation:

$$\left(\frac{p}{\gamma}\right)_{dy} = \left(\frac{H}{2}\right) \frac{\cosh \frac{2\pi}{L}(d + y_o)}{\cosh \frac{2\pi d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

The corresponding static pressures, acting at depths y_o, are

$$\left(\frac{p}{\gamma}\right)_s = \left(\frac{p}{\gamma}\right)^2_{dy} \frac{\pi}{L} \coth \frac{2\pi d}{L} + \left(\frac{p}{\gamma}\right)_{dy} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

If y is positive, the crest considered is above SWL and, therefore, becomes an additional component of the static pressure to be added to that given by Eq. 20.

[10] "Diques de Abrigo en Puertos," by Marciano Martinez Catena, *Revista de Obros Publicas, No. 2715*, July, 1941. (*Translation No. 44-1*, Waterways Experiment Station, Corps of Engrs., U. S. Army, Vicksburg, Miss., 1944.)

The d'Auria, Lira, and Iribarren solutions have been called the static-dynamic methods. The principal objections to these methods are: (*a*) They are based on the calculation of particle velocities at the crest of their elliptical orbits, which orbits cannot be followed at a vertical wall. (In other words, the methods assume that the breakwater does not disturb the wave movement, and the pressures are those that would be produced against a plane suddenly inserted at a wave crest); and (*b*) the theories do not explain observed phenomena at vertical breakwaters, such as the large quantities of water that pass over them without spray, and the considerable velocities that obtain at the base of vertical walls during wave attack.

In 1923 Victor Benezit[11] introduced a method that was the first to consider the effects of standing waves, or clapotis, on a vertical wall. The Benezit calculations were based upon the Gerstner theory.

"Clapotis" is a French term that refers to the phenomenon in which a series of progressive waves is reflected by a vertical surface perpendicular to the advancing waves and produces standing waves seaward of the structure. The equations of motion were obtained by superimposing opposite trains of progressive deep-water waves. The wave height of the clapotis was found to be $2H$, and the height of the orbital axis of these waves above SWL was assumed to be the same as that of the unobstructed waves (Eq. 2*b*). The pressure at a depth $d = L/2$ was assumed to be $\gamma d = \gamma L/2$. The derived pressure equation is

$$\frac{p}{\gamma} = z + \frac{\pi H^2}{2L}\left(e^{4\pi z/L} - \tfrac{1}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

These pressures are applied at distances r above and below the orbital axes, where

$$r = H e^{2\pi z/L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

This is twice the radius of the orbits described by particles of the unobstructed wave, as given by Eq. 3. The resulting pressure curves give the pressures on the seaward side of the breakwater for the crest and trough positions of the clapotis. If still water exists on the shoreward side of the structure, the net unbalanced pressure is obtained for both the crest and trough positions of the wave by subtracting the SWL hydrostatic pressure. Mr. Benezit assumed that $d = L/2$ regardless of the actual depth in which the breakwater was situated.

In 1928 George Sainflou[12] published a theory for calculating pressures on vertical breakwaters in finite depths. Mr. Sainflou superimposed two opposite trains of progressive elliptically trochoidal waves to obtain standing elliptically trochoidal waves, which type of motion, according to him, occurs in front of a vertical wall. The superimposing of elliptically trochoidal waves is not possible, rigorously, and further approximations other than those inherent in the progressive, shallow-water wave theory were necessary. In the resulting

[11] "Les Digues Maritimes Verticales," by Victor Benezit, *Annales des Ponts et Chaussees*, Paris, France, 1923.

[12] "Essai sur les Diques Maritimes Verticales," by George Sainflou, *Annales des Ponts et Chaussees*, Paris, France, 1928. (Translated copy on file in the office of the Div. Engr., Great Lakes Div., Corps of Engrs., U. S. Army, Chicago, Ill., 1938.)

wave movement (clapotis) a particle with coordinates (x_o, y_o) at rest describes an orbit represented by the equations

$$x = x_o + 2\,r \sin \frac{2\,\pi\,t}{T} \cos \frac{2\,\pi\,x_o}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23a)$$

and

$$y = y_o + \frac{4\,\pi\,r\,r'}{L} \sin^2 \frac{2\,\pi\,t}{T} + 2\,r' \sin \frac{2\,\pi\,t}{T} \sin \frac{2\,\pi\,x_o}{L} \quad \ldots\ldots\ldots (23b)$$

The maximum and minimum ordinates of the particle initially at rest at point x_o, y_o differ by the amount $4\,r' \cos \frac{2\,\pi\,x_o}{L}$. Hence, the wave height of the clapotis is $4\,r'$, or twice the height of the unobstructed wave. The crests of the clapotis orbits stand at a height $\frac{4\,\pi\,r\,r'}{L} + 2\,r'$ above the level y_o. The first term of this expression represents the maximum value of the mean level of the particle orbit above the particle at rest, or

$$h_z = \frac{4\,\pi\,r\,r'}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24a)$$

For the free surface, $y_o = 0$, $r' = \frac{H}{2}$, and $r = \frac{H}{2} \coth \frac{2\,\pi\,d}{L}$ (from Eq. 9), and

$$h_o = \frac{\pi\,H^2}{L} \coth \frac{2\,\pi\,d}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24b)$$

This is four times the corresponding value of h_o for the unobstructed wave, as given by Eq. 10. The Sainflou expression for pressure on a vertical wall is

$$\frac{p}{\gamma} = y_o \pm H \sin \frac{2\,\pi\,t}{T} \left[\frac{\cosh \frac{2\,\pi}{L}(d + y_o)}{\cosh \frac{2\,\pi\,d}{L}} - \frac{\sinh \frac{2\,\pi}{L}(d + y_o)}{\sinh \frac{2\,\pi\,d}{L}} \right] \ldots (25a)$$

When $\sin \frac{2\,\pi\,t}{T} = \pm 1$, Eq. 25*a* gives pressures for points situated a distance $2\,r'$ above and below the orbital axis $y_o + h_z$. When the clapotis is in a crest position on the wall, this equation represents a curve with a pressure of zero at height $H + h_o$ above SWL, and a pressure at the bottom ($y_o = -d$) of

$$\frac{p}{\gamma} = d + \frac{H}{\cosh \frac{2\,\pi\,d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25b)$$

When the clapotis is in a trough position, Eq. 25*a* represents a curve with a pressure of zero at a distance $(H - h_o)$ below SWL, and a pressure at the bottom of

$$\frac{p}{\gamma} = d - \frac{H}{\cosh \frac{2\,\pi\,d}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25c)$$

The resulting pressure curves can be approximated with an added factor of safety by straight lines, which makes for a very simple calculation of pressures on a vertical wall. In case still water exists on the shoreward side of the structure, the net unbalanced pressure is obtained, as mentioned previously, by subtracting the SWL hydrostatic pressure. If wave action exists on both sides of the breakwater simultaneously, the pressures are calculated for both sides of the structure and the more unfavorable algebraic combination of the two sets of pressures determines the net overturning moment.

In 1935, M. Gourret[13] published an interesting paper dealing with the clapotis in water of finite depth, in which he analyzed the approximations introduced by Messrs. Benezit and Sainflou. Mr. Gourret stressed the point that his method, as well as those of Messrs. Benezit and Sainflou, was applicable, theoretically, for waves of small height only, and that the approximations introduced became less allowable with decreasing depths. The equations derived by Mr. Gourret give results that are in fair agreement with those of Mr. Sainflou. The wave height of the clapotis was given as $2\,H$, as in the Sainflou method, and at the free surface, the height of the orbital axis above SWL was given as

$$h_o = \frac{\pi\,H^2}{L}\left(\tanh\frac{2\,\pi\,d}{L} - \coth\frac{4\,\pi\,d}{L}\right) + \frac{\pi\,H^2}{2\,L}\frac{\coth\dfrac{4\,\pi\,d}{L}}{\sinh^2\dfrac{2\,\pi\,d}{L}} \quad \ldots\ldots (26)$$

which gives values less than that obtained by the Sainflou method except for small values of d/L. Mr. Gourret determined the pressure on a vertical wall for the crest and trough positions of the clapotis. The resulting pressure curves can be replaced with straight lines without appreciable error. The pressure is zero at a height $(H + h_o)$ above SWL for the crest position, and zero at a distance $(H - h_o)$ below SWL for the trough position. The pressure at the bottom, at depth d, is

$$\frac{p}{\gamma} = d - \frac{\pi\,H^2}{L}\tanh\frac{2\,\pi\,d}{L} \pm \frac{H}{\cosh\dfrac{2\,\pi\,d}{L}} \quad \ldots\ldots\ldots\ldots (27)$$

the last term of Eq. 27 is positive when the clapotis is at the crest position and negative for the trough position.

In 1934 David A. Molitor[14] developed an empirical method of computing wave pressures on vertical breakwaters. By supplementing the wave-pressure data recorded by Mr. Gaillard,[7] he obtained a pressure curve the shape of which conforms to the envelope of maximum pressures at different elevations. According to the Molitor envelope curve, the pressure is zero at a height $H + \dfrac{4\,H^2}{L}$ above SWL and at a distance $H - \dfrac{2\,H^2}{L}$ below SWL. The maximum pressure, which was assumed to act at a height $h_o = \dfrac{2\,H^2}{L}$ above SWL,

[13] "Sur un Movement Approache de Clapotis, Application en Calcul des Digues Maritimes Verticales," by M. Gourret, *Annales des Ponts et Chaussees*, Paris, France, 1935.

[14] "Wave Pressure on Sea-Walls and Breakwaters," by David A. Molitor, *Proceedings*, ASCE, Vol. 60, May, 1934, p. 653.

was given as

$$\frac{p_m}{\gamma} = \frac{k}{2g}(C + V_{oc})^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which C and V_{oc} are given by Eqs. 12 and 11*b*, respectively. Mr. Molitor evaluated the factor k from Gaillard's observations on the Great Lakes and arrived at values for k ranging from 1.3 for a 30 miles per hr wind to 1.7 for a 70 miles per hr wind. For ocean storms, Mr. Molitor suggested a k-value of 1.8. These pressures and corresponding elevations, together with a pressure $p = 0.27\, p_m$ located at a distance $\frac{H}{4} - \frac{2H^2}{L}$ below SWL, determine the Molitor pressure curve. Mr. Molitor does not admit that there is pressure below the trough of the wave, which premise is contrary to both theory and experiment. The method by which he determined values for the empirical coefficient (k) is obscure.

Selection of Design Method (Nonbreaking Waves).—The results obtained using the different methods of computing wave pressures on vertical breakwaters, except the d'Auria method that was omitted, are shown in Table 1. The d'Auria theory is an oversimplification of the phenomena involved, and the overturning moments obtained are considerably higher than are obtained by the other theories. The values shown in Table 1 are calculated from the simplified versions of the various theories as shown in Fig. 2. The derivation of the pressure equations presented in Fig. 2 was based on the assumption that the theoretical pressure curves were straight lines in each case. Although liberties were taken with the original theories in devising the simplified methods of calculation, these curves are of sufficient accuracy for comparing the different methods, and, in so far as the original theories are adequate, the simplified methods are believed to give results within the range of accuracy required for actual design of vertical breakwaters. The diagrams of Fig. 2 show the magnitude and distribution of wave pressures from which the maximum (wave at crest) net overturning moments shoreward can be calculated, assuming SWL hydrostatic pressure on the shoreward side. For all methods based upon the formation of the clapotis, similar diagrams could be prepared to show net pressures seaward when the wave was at a trough position, and for net unbalanced pressures when wave action existed on both sides of the breakwater. The methods explained in preceding paragraphs can be placed in three categories: (1) The static-dynamic d'Auria, Lira, and Iribarren methods; (2) the standing-wave or clapotis methods of Messrs. Benezit, Sainflou, and Gourret; and (3) the Molitor empirical method. Recalling the objections to the static-dynamic theories in general, and to the d'Auria method in particular, and noting from Table 1 that the Lira method gives overturning moments much smaller than any other method, it is seen that the Iribarren method is the only one of this type that could be considered for use. In the standing-wave methods, that of Mr. Benezit should be discarded, although it gives pressures nearly as large as the Sainflou method, since it deals only with waves in deep water. By the process of elimination the conclusion has been reached that the Iribarren, Sainflou, and Gourret methods are the theories that should be considered for use in calculating wave forces on vertical breakwaters. Until

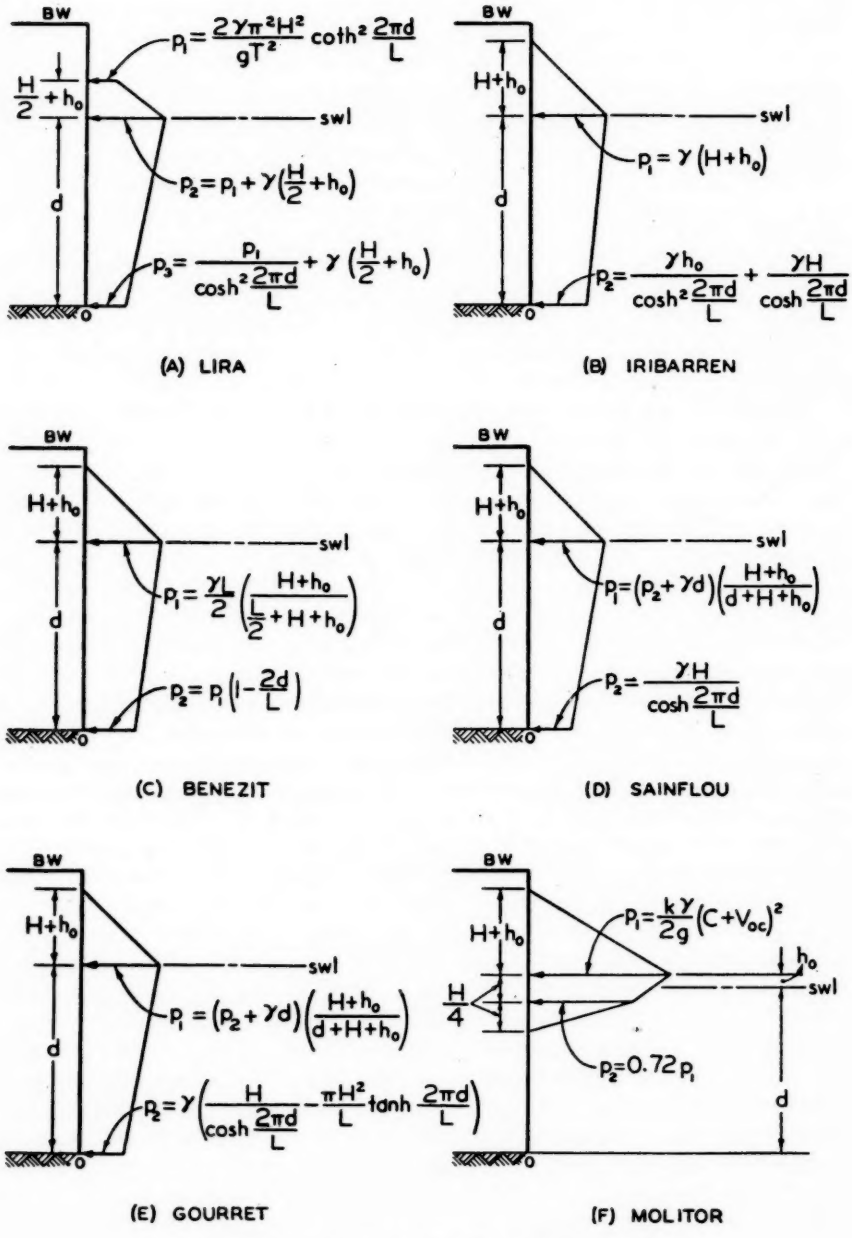

FIG. 2.—WAVE PRESSURE DIAGRAMS, MODIFIED THEORIES

TABLE 1.—WAVE PRESSURE THEORIES—COMPARISON OF RESULTS

ITEM					
Ratio H/L	Ratio d/L	h_o (ft)	Crest above still water level (ft)	Pressure per unit length P_t (lb per ft)	Overturning moment M_o (ft lb per ft)
(*a*) J. LIRA (1926)					
0.057	0.252	0.6	6.6	28,700	855,000
0.057	0.143	1.0	7.0	20,300	362,900
0.020	0.088	0.7	6.7	28,700	826,200
0.020	0.050	2.0	8.0	22,800	401,900
(*b*) R. IRIBARREN (1938)					
0.057	0.252	0.6	12.6	33,700	1,168,000
0.057	0.143	0.8	12.8	25,200	494,600
0.020	0.088	0.4	12.4	42,800	1,306,500
0.020	0.050	0.6	12.6	28,000	518,500
(*c*) V. BENEZIT (1923)					
0.057	0.252	0.5	12.5	32,100	1,067,000
0.057	0.143	0.5	12.5	22,300	434,400
0.020	0.088	0.2	12.2	39,800	1,218,500
0.020	0.050	0.2	12.2	25,300	469,800
(*d*) G. SAINFLOU (1928)					
0.057	0.252	2.3	14.3	33,600	1,174,000
0.057	0.143	3.0	15.0	25,800	528,000
0.020	0.088	1.5	13.5	43,800	1,366,000
0.020	0.050	2.5	14.5	29,400	571,900
(*e*) M. GOURRET (1935)					
0.057	0.252	0.0	12.0	25,400	901,000
0.057	0.143	0.4	12.4	20,600	406,000
0.020	0.088	0.8	12.8	41,700	1,291,000
0.020	0.050	5.5	17.5	33,000	686,900
(*f*) D. MOLITOR (1934)					
0.057	0.252	1.4	14.7	18,300	1,036,600
0.057	0.143	1.4	14.7	16,800	565,000
0.020	0.088	0.5	13.0	28,100	1,563,000
0.020	0.050	0.5	13.0	20,700	673,300
(*g*) MODEL DATA (*a*) (1945)					
0.057	0.252	−3.0	9.0	25,600	821,700
0.057	0.143	2.5	14.5	24,500	500,000
0.020	0.088	4.0	16.0	52,500	1,670,900
0.020	0.050	16.0	28.0	50,100	1,185,400

(*a*) Waterways Experimentation Station, Vicksburg, Miss.

such time when the science of wave action has been placed on a sounder basis, it is believed that the Sainflou and Gourret methods should be used to calculate moments, and the one adopted that shows the worse effects on stability. Also, these methods should be used to obtain the crown height of the wall, adopting the one that gives the maximum value. The Iribarren method is very good, but it is rather complicated in application, and, too, one or the other of the methods selected gives results greater than that of the Iribarren formula in all cases. The empirical method of Mr. Molitor gives results in fair agreement with the better theories, but the value of this method for design depends on a judicious selection of coefficients corresponding to different wind velocities and durations. (Values of k used for purposes of comparing the Molitor method with the other wave-pressure theories were 1.3, 1.4, 1.5, and 1.6 for wave periods of 6.7, 7.6, 15.2, and 19.7 sec, respectively. These wave periods correspond to the d/L ratios in Table 1 of 0.252, 0.143, 0.088, and 0.05.) The Molitor method should be used with caution and only in locations for which accurate coefficients have been established.

Adequacy of Methods Selected.—E. Coen-Cagli,[15] in 1936, criticized the methods of Mr. Sainflou and others adhering to the clapotis theory, on the basis that standing waves are not formed at a vertical wall unless wave heights are small. The clapotis theories assumed small wave heights, yet it was claimed that reasonably accurate results were obtained by application of these theories when wave heights became appreciable. Mr. Coen-Cagli declared that natural storm waves retain the essential characteristics of unobstructed trochoidal waves and, consequently, cause pressures greater than the clapotis, and that the ratio of pressures that actually obtain, to the pressures calculated from the clapotis theories, increases with decreasing d/L-ratios. In proving his contentions Mr. Coen-Cagli cited prototype and model investigations of wave action on breakwaters in the ports of Genoa, Italy, and Algiers, Algeria. From these experiments the following conclusions were drawn and represent the ideas of Mr. Coen-Cagli as to the magnitude and distribution of pressures on vertical breakwaters: (*a*) The larger storm waves are not transformed in the clapotis movement, but retain the essential characteristics of unobstructed waves although the orbital motion is changed; (*b*) the total force exerted is greater than that of the clapotis; (*c*) the maximum pressure is near and slightly below SWL and is approximately equal to γH; and (*d*) pressure decreases linearly from SWL to a point about $\frac{2}{3} H$ above SWL, whereas, below SWL the pressure decreases slowly with depth, the rate of decrease becoming less with smaller values of H/L. Mr. Coen-Cagli did not present a mathematical expression for wave force, but recommended that model tests be conducted for each breakwater problem. His conclusions were based on prototype observation and model tests and are borne out by the results of his experiments. In the model tests, however, waves were used that formed a clapotis, as well as others of the same dimensions that did not. The manner whereby the latter feat was accomplished is not understood. Furthermore, if the Sainflou equations are used to calculate pressures, using the same dimensions of H, L,

[15] "L'Action des Lames de Tempete sur les Digues Maritimes a Paroi Verticale," by E. Coen-Cagli, *Le Genie Civil*, Vol. 109, No. 9, August, 1936. (*Translation No. 43-36*, Waterways Experiment Station, Corps of Engrs., U. S. Army, Vicksburg, Miss., 1943.)

and d as were used by Mr. Coen-Cagli in his model tests, the calculated pressures agree more closely with the results of tests in which the waves did not form a clapotis. Therefore, the conclusions of Mr. Coen-Cagli can be viewed with some suspicion. On the other hand, model tests conducted at the Waterways Experiment Station,[16] Corps of Engineers, United States Army, Vicksburg, Miss. (hereinafter called the Waterways Experiment Station), seem to bear out the contentions of Mr. Coen-Cagli. The results of Waterways Experiment Station tests are shown in Table 1, and Fig. 3 shows a comparison of the test results with overturning moments about the base calculated from the Sainflou

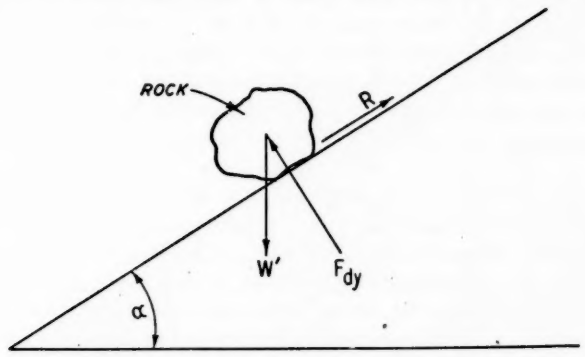

FIG. 3.—RESULTS OF TESTS AT U. S. WATERWAYS EXPERIMENT STATION

theory. In this figure the ratio M_m/M_s (model-test moments divided by moments calculated from the Sainflou theory) is plotted against the ratio d/L. It can be seen that the ratio M_m/M_s increases as the d/L-ratio decreases, which agrees with the Coen-Cagli contentions. It is believed, therefore, that the ideas of Mr. Coen-Cagli should be taken seriously, and further experiments carried out to verify the relation between actual forces exerted by waves on vertical walls and those calculated by the Sainflou theory. If a simple relation between theoretical and actual forces (as indicated by Fig. 3) could be proven, it is believed that the design of vertical breakwaters situated in depths of water sufficient to prevent breaking waves would be possible within the degree of accuracy of design wave selection.

Pressures Due to Breaking Waves.—If vertical breakwaters are situated where the bottom slopes seaward, and in depths approximately equal to the wave height, the waves are no longer reflected but are orbitally destroyed and break on the structure with resulting shock pressures much larger in magnitude than those of the clapotis-type waves. This phenomenon has been studied by Ralph A. Bagnold[17] in 1939 and by J. R. Morison[18] in 1948, but their analysis and experiments were not sufficiently extensive to furnish final solution to the problem. Mr. Bagnold found that all breaking waves do not result in

[16] "Model Tests of Portable Breakwaters for D-Day Invasion Harbors," by Robert Y. Hudson, *Civil Engineering*, Vol. 15, 1945, p. 405.

[17] "Interim Report on Wave-Pressure Research," by Ralph Alger Bagnold, *Journal*, Institution of Civil Engineers, Vol. 12, 1939, p. 202.

[18] "Wave Pressures on a Vertical Wall," by J. R. Morison, *Technical Report-HE-116-298*, Dept. of Eng., Univ. of California, Berkeley, Calif., December, 1948.

shock pressures, because a critical system of conditions is necessary before the phenomenon of high shock pressures can occur. According to Mr. Bagnold, the critical condition necessary is the trapping of a thin cushion of air between the folding front of a breaking wave and the wall. The formula he proposed for the maximum pressure developed by a breaking wave is

$$\frac{p_m}{\gamma} = \frac{2.7}{g} C^2 \frac{m}{b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

in which m is the length of water column brought to rest by the air cushion and b is the thickness of that cushion. Mr. Bagnold estimated the value of m to be about $0.2\,H$. Mr. Morison estimated the value of b to be about 0.02 ft for a particular model wave. The formula is dimensionally homogeneous and can be used in any system of consistent units.

Further Investigation.—For vertical breakwaters situated in depths that prevent the breaking of waves, it is believed that the Sainflou equations should be used and the resulting pressures or moments modified according to corrective coefficients such as those indicated by Fig. 3. However, it is evident that four data points are not sufficient to determine the magnitude of the desired corrective coefficients, and it is desirable that further tests be made covering a larger range of the variables involved. All theories and tests referred to in preceding paragraphs concern waves approaching normal to the breakwater alinement. It is not considered necessary to perform extensive investigations for waves attacking at angles of incidence greater than zero degrees, since J. Larras[19] has shown, both by theoretical analysis and model tests, that wave pressure on a vertical wall does not vary appreciably the angle of incidence.

Future investigation of the phenomenon of shock-type pressures caused by breaking waves should be concerned with the following aspects of the problem: (*a*) Definite delineation of the critical conditions necessary for the occurrence of shock pressures of maximum intensity over the complete range of breaking waves as a function of the degree of break; (*b*) modification and extension of the Bagnold equation to a usable form; (*c*) delineation of the area over which the high pressures may act simultaneously; and (*d*) determination of the frequency of occurrence of shock pressures of high intensity.

Rubble-Mound Breakwaters

The use of rubble breakwaters for protecting harbors and shore-line structures is indicated for localities in which the water is not very deep and an adequate quantity of suitable rock is available at reasonable cost. Although a greater volume of rock is required to construct a rubble breakwater than is required for construction of a vertical-wall, gravity-type structure, the unit cost of placing the rubble is less, and, if a good quarry is available in the vicinity of the site, the unit cost of material is less. A rubble mound never fails completely under the attack of waves greater than the selected design wave. As rocks are displaced from a rubble mound by wave action, the breakwater tends to become more stable. Also, repair of the damage is relatively cheap. On

[19] "La Resistance des Jettees Veriticles aux Houles Obliques," by J. Larras, *Annales des Ponts et Chaussees*, August, 1937. (Translated copy on file in the Office of the Chief of Engrs., Dept. of the Army, Washington, D. C.)

the other hand, the failure of a vertical-wall breakwater is usually conclusive and entails more expensive reconstruction.

Design Methods.—Rubble breakwaters are usually designed by rule-of-thumb based upon past experience. One philosophy of design, by no means without merit, is to obtain the largest rock available at a reasonable price and construct the breakwater with the steepest slope possible using available equipment. If and when rocks are displaced from the section by waves, the structure is repaired by dumping more rock. This method takes advantage of the fact that the removal of rock by wave action from the upper portions of a rubble mound results in flatter slopes and a more stable section. If it were not for the understandable reluctance of engineers to approve the design of a structure that is expected to fail before a successful design is obtained, it is believed that this method of designing rubble breakwaters would be more prevalent. What is needed, of course, is a method of design that allows the use of known factors of safety in order that the most economical type breakwater can be selected.

Some use is made of semirational and empirical formulas, but there is no formula available for the design of rubble breakwaters that can be used with assurance. Mr. Iribarren[20] developed a formula for the design of rubble-mound breakwaters in 1938. Harris Epstein and F. C. Tyrrell, M. ASCE, derived a formula similar to that of Mr. Iribarren, although developed from different concepts, in 1949.[21] The formula proposed by Mr. Iribarren is

$$W = \frac{K \gamma_r H^3}{(\gamma_r - 1)^3(\cos \alpha - \sin \alpha)^3}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

in which W is the weight of individual cap rock in kilograms; K is a coefficient = 15 and 19 for natural and artificial rock, respectively; H is wave height in meters; γ_r is the specific weight of cap rock in metric tons per cubic meter; and α is the angle, measured from horizontal, of the breakwater slope. The coefficient of friction between rock was assumed to be unity. In this formula K is not dimensionless. The values of K given above were determined by Mr. Iribarren from observations of actual breakwaters. A more general form of Eq. 30, which is dimensionally homogeneous and includes the coefficient of friction, is

$$W = \frac{K' \gamma_r \gamma_f^3 u^3 H^3}{(\gamma_r - \gamma_f)^3(\mu \cos \alpha - \sin \alpha)^3}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

in which K' is an undetermined dimensionless coefficient, γ_r is the specific weight of cap rock, γ_f is the specific weight of liquid in which the rock is submerged, and μ is the effective coefficient of friction between rock. Since Eq. 31 is dimensionally correct, it is equally valid for any system of units. By comparison of Eqs. 30 and 31, and by dimensional considerations, it can be shown that the values of K' corresponding to $K = 15$ and $K = 19$ are $K' = 0.015$ and $K' = 0.019$, respectively.

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[20] "Translation of a Formula for the Calculation of Rock-Fill Dikes," by Ramon Iribarren Cavanilles, *Technical Report-HE-116-295*, Dept. of Eng., Univ. of California, Berkeley, Calif., August, 1948.

[21] "Design of Rubble-Mound Breakwaters," by Harris Epstein and F. C. Tyrrell, Section 2, Communication 4, XVIIth International Congress of Navigation, Lisbon, Portugal, 1949.

Derivation of the original Iribarren formula and that of Eq. 31 was based upon the assumption that dynamic forces tending to displace rock from the breakwater slope are proportional to wave height, area of rock over which the forces act, and the specific weight of the liquid ($F_{dy} = k\,A\,\gamma_f\,H$). Also, the analysis is based on an assumed force diagram for a single rock, as shown in Fig. 4. For equilibrium, the friction force R must balance the down-slope component of the submerged rock weight W'. The dynamic force (F_{dy}) is assumed to act upward, perpendicular to the breakwater slope. This assumption is based on the premises that: (1) The waves break on the structure and

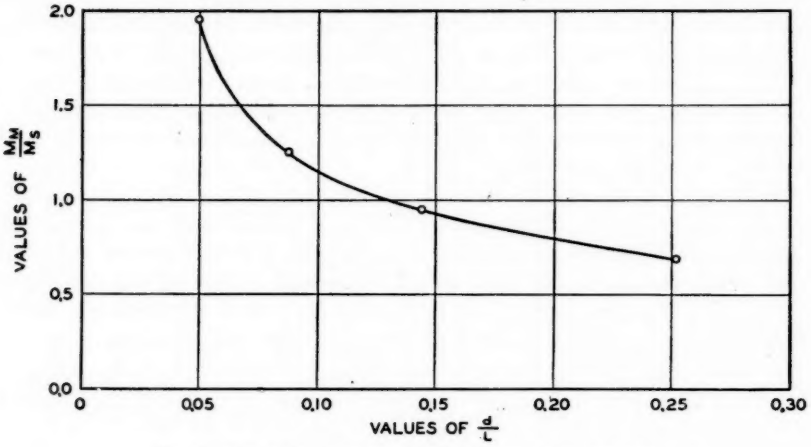

FIG. 4.—FORCE DIAGRAM FOR RUBBLE-MOUND BREAKWATER

direct jets of water downward perpendicular to the slope; and (2) at the beginning and end of the splashes the jets create forces opposite in direction to the flow of water in the jets. Although the latter premise is probably correct generally, the assumption that a wave makes a complete break and directs a jet downward perpendicular to the slope is open to criticism. Experiments at the Waterways Experiment Station indicate that waves at a breakwater tend to do one of three things: (*a*) They may execute a complete break with the direction of the jet approximately perpendicular to the slope; (*b*) they may be reflected and establish a standing wave system seaward of the structure; or (*c*) they may execute a partial break, with the resulting jet action poorly defined, and with a portion of the wave energy reflected. The formula can also be questioned because it does not take into account the inherent stability or instability of the attacking waves (which is a function of H, L, and d), voids in the rubble, width of breakwater near SWL, and rock shape and effective roughness. The effects of these variables, which are not included in the formula, and the effects of inaccuracies in the basic assumptions must be contained in the coefficient K'. If the Iribarren formula is to be made sufficiently accurate for design purposes, over the range of variables met in practice, it will be necessary to determine, either experimentally or from a large number of prototype observations, the important variables contained in K', and variations of K' with variations of these important variables.

The formula proposed by Messrs. Epstein and Tyrrell is

$$W = \frac{R_t \gamma_r \gamma^2_f \cos^3 \alpha H^3}{(\gamma_r - \gamma_f)^3(\mu \cos \alpha - \sin \alpha)^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which R_t is a dimensionless coefficient. Derivation of this formula was based on a force diagram similar to that of Mr. Iribarren (Fig. 4), except that a tangential force, attributed to movement of the water, was added. It was assumed that the vertical component of the dynamic pressure on the rock is proportional to the square of the vertical component of the orbital velocity of the wave motion, and that the horizontal component of dynamic pressure is proportional to the product of the wave celerity and the horizontal component of the orbital velocity. Also, the formula was derived assuming that the waves at the breakwater do not break. Although it is difficult to interpret rationally the assumptions upon which the Epstein-Tyrrell formula is based, surprisingly, the formula obtained is the same as that of Iribarren if $R_t = \frac{K' \mu^3 \gamma_f}{\cos^3 \alpha}$. Messrs. Epstein and Tyrrell also developed a formula for R_t in terms of α, μ, and the d/L ratio, but the formula is rather complicated and involves four additional unknown coefficients, some of which are probably variables. Evaluation of the coefficients in R_t, therefore, would involve about the same amount of work as that in evaluating the coefficient K' of the Iribarren formula.

Model Tests in Progress (1950).—Tests of small-scale rubble breakwaters were being conducted in 1950 by the Waterways Experiment Station for the Bureau of Yards and Docks, Department of the Navy, in which the coefficients of the Iribarren and Epstein-Tyrrell formulas are being evaluated. Tests completed (1950) have been concerned with evaluating the variables contained in K' of the Iribarren formula. The testing program will provide data for cap rock of three sizes and corresponding void ratios, one shape factor, and four specific gravities. One depth of water and three wave periods are being used, corresponding to d/L ratios of 0.16 to 0.38. Although the testing program is incomplete, the results obtained to date are instructive and indicate the type and extent of experiments needed to furnish adequate coefficients for the Iribarren formula. Some of the test results are shown in Fig. 5, in which K' is plotted against the d/L ratio, with breakwater slope as a parameter. The coefficient of friction (u) used in calculating K' was obtained by taking an average of the angles of repose determined by dumping the rock under water and placing the rock by hand. When dumped, the angle of repose was found to be 45°. When placed by hand, as was done for the model tests, the rock could be made to stand at an angle slightly greater than 50°. The coefficient used (1.09) is the tangent of a 47° 30′ angle. It can be seen that K' is not constant but varies with both d/L and α. The testing program had not advanced sufficiently at the time of preparation of this paper to show whether these are the two most important of the variables contained in K'.

Further Investigations.—Other experiments that are thought necessary to complete the testing program relative to the Iribarren formula are indicated by the dashed lines of Fig. 5. The range of d/L-ratios should be extended to

include a minimum value of the ratio of about 0.05 and a maximum value of 0.5. Tests to date have been performed with the waves approaching normal to the structure. Tests using other angles of incidence should be made and are included in the testing program authorized by the Bureau of Yards and Docks. The tests in 1950 were conducted in such a manner that no overtopping of the breakwater crown by attacking waves was permitted, and the greatest wave height that would not cause displacement of rock from the face

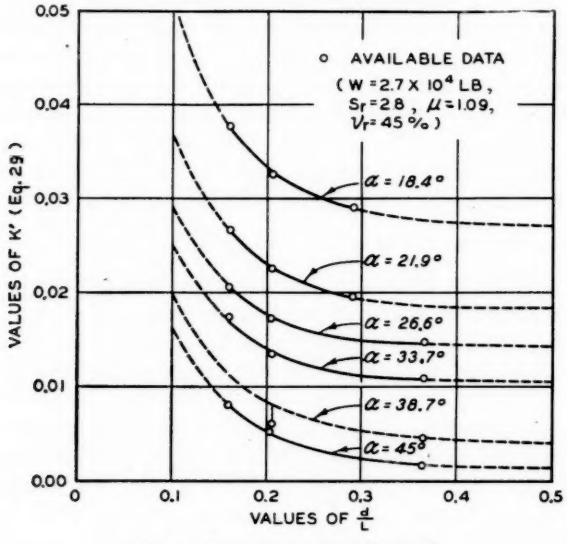

FIG. 5.—VALUES OF COEFFICIENT K'

slope was determined. It has been found, however, that if a slight amount of damage, insufficient to reduce appreciably the efficiency of the breakwater, is allowed, the design waves can be increased several feet in height. Therefore, compilation of data showing the effects on K' of allowing slight overtopping of, and damage to, the breakwater crown should prove invaluable to the design engineer, giving him considerably more latitude in selecting the most economical breakwater for a given degree of exposure to wave action. Tests should also be performed to isolate the effects of rock shape factor.

The outlined testing program is very general in character and is designed primarily to evaluate the coefficient K' in the Iribarren formula. There are many questions of design that are left unanswered by these experiments. However, tests of rubble breakwaters, even with small-scale models, are very expensive and time consuming. It is believed, therefore, that the general testing program should be completed first, leaving tests of particular breakwater types and problems of a less general nature for future study.

CONCLUSIONS

Vertical-Wall Breakwaters.—Until the science of wave action on vertical walls has been placed on a sound basis, the theories of Messrs. Sainflou and

Gourret should be utilized for the design of breakwaters situated in depths that do not cause breaking waves. Experiments should be performed to obtain coefficients that will bring calculated pressures in line with actual measured values. Experiments should also be performed to determine the magnitude, distribution and frequency of occurrence of shock-type pressures caused by breaking waves.

Rubble-Mound Breakwaters.—Because of the complexity of the phenomena whereby waves attack a mound of rock, and the many variables involved in waves and in the characteristics of rock and rock mounds, it is not believed necessary to develop a more accurate theoretical basis than that of the Iribarren formula. However, before rubble breakwaters can be designed with known factors of safety, further experimental data will be required to evaluate the coefficient (K') of this formula.

Acknowledgments

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Appendix.—List of Symbols

The following letter symbols introduced in this paper are assembled for the convenience of the discussers:

- b = thickness of air cushion, in Eq. 29
- C = wave celerity
- d = depth of water measured from SWL
- e = 2.3183, base of natural logarithms
- F_{dy} = dynamic force
- g = gravitation acceleration
- γ = specific weight (wt/unit volume)
- h_z = distance of a particle's orbital axis above its plane of rest
- h_o = distance of a surface particle's orbital axis above SWL
- H = wave height, trough to crest
- k = cofficient in Eqs. 16 and 28
- K = coefficient in Iribarren's original formula (Eq. 30)
- K' = dimensionless coefficient in the generalized form of Iribarren's formula (Eq. 31)
- L = wave length
- m = length of water column brought to rest by air cushion (Eq. 29)
- M = net overturning moment (shoreward) about the base
- p = pressure intensity

P_t = total pressure per unit length of breakwater
r = horizontal radius of orbital ellipse
r' = vertical radius of orbital ellipse (in deep water $r = r'$; that is the orbits are circles)
SWL = still-water level, level of free surface at rest
T = wave period
t = time elapsed by a particle in wave motion
μ = effective coefficient of friction, rock on rock
V = orbital velocity of a particle
W = weight of individual cap rock in Eqs. 30, 31, and 32
x = abscissa of particle in wave motion
x_o = abscissa of particle at rest
y = ordinate of particle in wave motion
y_o = ordinate of particle at rest
z = distance from orbital axis of surface particle to orbital axis of particle originally at rest at depth y_o
α = angle measured from horizontal of rubble breakwater slope on sea or lake side of the structure